A great probabilist: Catherine Doléans-Dade

B. Hajek

Department of Electrical and Computer Engineering
and the Coordinated Science Laboratory
University of Illinois at Urbana-Champaign

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Abstract:

A brief presentation of some of the remarkable technical contributions of Catherine Dolèans-Dade.

Catherine Doléans-Dade

Overview of technical contributions

- Catherine Doléans completed her graduate study under the direction of P. A. Meyer at the University of Strasburg in the late 1960's.
- Main publications appeared 1966-1979.
- Deepest work concerned the theory of predictable compensators for continuous-time random processes.
- Significant contributions to the calculus of martingles, including a general change of variables formula, a theorem on stochastic differential equations, and exponential processes of semimartingales.
- Work is relevant today, especially for models mixing discrete and continuous aspects, such as in modeling timing channels in biology or financial systems, working with likelihood ratios in such contexts.

A bit of background

Suppose $B = (B_k : k \geq 0)$ is a submartingale relative to a filtration $(\mathcal{F}_k : k \geq 0)$:

- So B_k is $\mathcal{F}_k$ measureable $\forall k$, (i.e. B is $\mathcal{F}.$ adapted), and
- B has positive drift: $E[B_{k+1} - B_k | \mathcal{F}_k] \geq 0$.

The *predictable compensator, A*, of B satisfies

- So B_k is $\mathcal{F}_{k-1}$ measureable $\forall k$, (i.e. A is $\mathcal{F}.$ predictable), and
- $B - A$ is a martingale

J.L. Doob discussed compensators in discrete time, and conjectured that similar compensators should exist in some generality for continuous time.

P. A. Meyer, who visited Doob at Illinois, proved the "Doob-Meyer decomposition theorem" to address this.

But the definition/characterization of A, Meyer's natural increasing processes, was not satisfactory.

Catherine Dolèan's work on predictable projections, and the measure she defined on predictable sets, led to the accepted final version of the decomposition theorem.

The usual conditions

- Assume $(\Omega, \mathcal{F}, P)$ is complete (subsets of events with probability zero are events)
- Assume filtration of σ-algebras $\mathcal{F}_\bullet = (\mathcal{F}_t : t \geq 0)$ is
  - right-continuous, and
  - each $\mathcal{F}_t$ includes all zero-probability events.
- Thus martingales, supermartingales, and submartingales have càdlàg (right continuous with finite left limits) versions. We assume in these slides such versions are used, without further explicit mention.

Doléans measure [1]

Connected with the modern version of the Doob-Meyer decomposition theorem. The measure is at the core of predictable projections.

- A random process X is a function of two variables: $(X(t,\omega) : (t,\omega) \in \mathbb{R}_+ \times \Omega)$.
- The *predictable sets* consist of the σ-algebra of subsets of $\mathbb{R}_+ \times \Omega$ generated by left-continuous adapted random processes.
- The predictable projection of a submartingale B is a predictable process A such that $A_\tau = E[B_\tau | \mathcal{F}_{\tau-}]$ for all finite predictable stopping times τ.
- The Dolèans measure for a submartingale B is the measure μ on the predicable σ-algebra such that $\mu[[0,\tau]] = E[B_\tau]$ for all bounded predictable stopping times τ.

Dolèans-Meyer change of variable formula [2, 3]

Let F by a twice continuously differentiable function and let X be a semimartingale. Then $F(X)$ is a semimartingale and:

$$F(X_t) = F(X_s) + \int_s^t F'(X_{u-}) dX_u + \sum_{s<u\leq t} \left(F(X_u) - F(X_{u-}) - F'(X_{u-}) \triangle X_u \right) + \frac{1}{2} \int_s^t F''(X_u) d[X,X]_u^c \tag{1}$$

Dolèan-Dade exponential [4]

Let X be a semimartingale, and let $Z = \mathcal{E}(X)$ be the solution of $Z_t = 1 + \int_0^t Z_{s-} dX_s$. Then

$$Z_t = \exp(X_t - X_0 - \frac{1}{2} < X^c, X^c >_t) \prod_{s \leq t} (1 + \triangle X_s) \exp(-\triangle X_s)$$

- If X is a local martingale then so is $\mathcal{E}(X)$.
- If X and Y are semimartingales, $\mathcal{E}(X + Y) = \mathcal{E}(X)\mathcal{E}(Y)$.

A dream course on martingale calculus

In Spring 1976, Professor Doléans-Dade presented a one semester course on martingale calculus, including

- Basic martingale inequalities, càdlàg versions, classification of stopping times
- theory of analytic pavings and predictable projections and compensators
- change-of-variable formula
- stochastic differential equations for martingales
- change of measures
- semimartingale exponentials

I took this course with Edwin A. Perkins, now Professor and Canada Research Chair in Probability, University of Brittish Columbia. I never learned as much in any other course.

In Memoriam: Catherine Doléans-Dade[1]

Catherine Doléans-Dade died on Sept. 19, 2004, after a long struggle with cancer. She was a long time member of the University of Illinois probability group, as well as the wife of mathematics professor Everett Dade. In the theory of martingales she was known for what is now called the Doléans measure. Catherine Doléans first came to the U of I Mathematics Department as a visiting graduate student under a Fulbright grant in 1967-68. After receiving her Doctorat d'Etat from the University of Strasbourg, France, in 1970, she returned here with her husband and first child in 1971. She was an Assistant Professor in the Mathematics Department from 1971 to 1979, when she resigned to take care of her two growing children. From 1981 until her death she was an Adjunct Associate Professor in the Mathematics Department. At various times during this period she was an editor for the *Annals of Probability*, and for the *Illinois Journal of Mathematics.* After her children were grown, she taught from time to time for the U of I Statistics Department, for Home Hi, and for University High School, where she worked until last spring, when she became too ill to continue.

[1] Source: http://www.math.uiuc.edu/People/memoriam_doleans-dade.html

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