Economics 468

October 19, 2020 **R. Davidson**

Midterm Examination

Your completed exam should be uploaded to myCourses by 13.00 on October 19. Please submit two files per student: one, which should be a PDF file, with your written answers, and another, which can be a simple text file, with your computer code. These files must be all your own work. You may make use of whatever non-human resources you wish, but you must not ask for or receive any help from any other person.

All students in this course have the right to submit in English or in French any written work that is to be graded.

Tou(te)s les étudiant(e)s qui suivent ce cours on le droit de soumettre tout travail écrit en français ou en anglais.

Academic Integrity statement [approved by Senate on 29 January 2003]:

McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures.

L'université McGill attache une haute importance à l'honnêteté académique. Il incombe par conséquent à tou(te)s les étudiant(e)s de comprendre ce que l'on entend par tricherie, plagiat et autres infractions académiques, ainsi que les conséquences que peuvent avoir de telles actions, selon le Code de conduite de l'étudiant(e) et des procédures disciplinaires.

Answer all four questions in this exam.

Faites tous les quatre exercices de cet examen.

1. For this question, the notation $\boldsymbol{P_Z}$ signifies the orthogonal projection on to the span of the columns of the matrix $\boldsymbol{Z}$. Similarly, $\boldsymbol{M_Z}$ is the complementary projection: $\boldsymbol{M_Z} = \mathbf{I} - \boldsymbol{P_Z}$.

Let $\boldsymbol{\iota}$ be an n-vector of 1s, and let $\boldsymbol{X}$ be an $n \times 3$ matrix, with full rank, of which the first column is $\boldsymbol{\iota}$. What can you say about the matrix $\boldsymbol{M_\iota X}$? What can you say about the matrix $\boldsymbol{P_\iota X}$? What is $\boldsymbol{M_\iota M_X}$ equal to? What is $\boldsymbol{P_\iota M_X}$ equal to?

2. You will find the data to be used for this question at the URL:

https://russell-davidson.arts.mcgill.ca/data/e468.midterm.20.dat

There are 100 observations, representing 25 years of quarterly data. The first column contains the observations for the variable Y, for disposable income. The second column is for C, consumption expenditure.

Regress C on a constant and Y, and give the interpretation of the estimated coefficients under the assumption that the model is correctly specified.

Generate four seasonal indicator variables $\boldsymbol{s}_1$, $\boldsymbol{s}_2$, $\boldsymbol{s}_3$, and $\boldsymbol{s}_4$. Now regress C on these four seasonal variables and Y. Give the interpretation of the results of this new regression. Explain why there is no constant in the set of regressors.

Next, regress C on the seasonal variables only, and save the residuals. Do the same for Y. Regress the residuals from C on the residuals from Y, with no other regressor at all. Explain what all this means, and show that the parameter estimate from this last regression is equal to one of the estimates from an earlier regression.

Now generate three seasonal dummies, as follows:

$$\boldsymbol{s}_1' = \boldsymbol{s}_1 - \boldsymbol{s}_4, \quad \boldsymbol{s}_2' = \boldsymbol{s}_2 - \boldsymbol{s}_4, \quad \boldsymbol{s}_3' = \boldsymbol{s}_3 - \boldsymbol{s}_4.$$

Regress C on a constant, these three new variables, and Y. Interpret the estimated coefficient of the constant, and show that its numerical value is compatible with your earlier results.

3. Consider the linear regression $\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}$, with our usual notation. Let $\boldsymbol{P_X}$ denote the orthogonal projection on to the span $\mathcal{S}(\boldsymbol{X})$ of the regressors, and let the leverage measure h_t be the tt-diagonal element of $\boldsymbol{P_X}$. The unit basis vector $\boldsymbol{e}_t$ is the vector all of whose elements are zero except for element t, which is equal to 1. Show that the h_t is the square of the cosine of the angle between the unit basis vector $\boldsymbol{e}_t$ and its projection on to the invariant subspace of $\boldsymbol{P_X}$.

4. Let the scalar random variable Y_n have CDF

$$F_n(x) = \begin{cases} 0 & \text{for } x < 0, \\ nx & \text{for } 0 \le x \le 1/n, \\ 1 & \text{for } x > 1/n. \end{cases}$$

Y_n is said to have the uniform distribution $\mathrm{U}(0, 1/n)$, since its density is constant and equal to n on the interval $[0, 1/n]$, and zero elsewhere.

Show that the sequence $\{Y_n\}$ converges in distribution. What is the limiting CDF F_∞? Show that F_∞ has a point of discontinuity at 0, and show also that $\lim_{n\to\infty} F_n(0) \neq F_\infty(0)$. (CDF means "cumulative distribution function".)