

Wavelet Analysis of Commodity Price Behavior

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Abstract

We propose a form of semi-nonparametric regression based on wavelet analysis. Traditional time series methods usually involve either the time or the frequency domain, but wavelets can combine the information from both of these. While wavelet transforms are typically restricted to equally spaced observations an integer power of 2 in number, we show how to go beyond these constraints. We use our methods to construct “patios” for 21 important international commodity price series. These graph the magnitude of the variations in the series at different time scales for various subperiods of the full sample.

Keywords Wavelets, Semi-nonparametric Regression, Commodity Price Behavior, Multi-resolution Analysis.

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1. Introduction

In this study we propose a form of semi-nonparametric regression based on wavelet analysis. Traditional time series analyses rely on methods that involve either the time or the frequency domain. But wavelet transforms permit an analysis that combines both time and frequency information, the latter in terms of levels of time resolution. Usually wavelet transforms are used only with equally spaced observations whose number is an integer power of two. In this paper, we show how to go beyond these constraints. In an empirical application, we use our methods in order to construct “patios” for some important international commodity price series. These patios show the magnitude of the variations in the series at different time scales for different subperiods of the full sample.

In the next section, we begin with a brief review of past econometric findings concerning commodity price behavior. Then, in section 3, we present a discussion of multi-resolution analyses, in particular their implementation in terms of wavelets. Section 4 contains a description of how we extend traditional multi-resolution analyses to the context of nonparametric regression, and in section 5, we present the results of the application of our methods to some commodity price series. Section 6 endeavors to provide an economic interpretation of the results, and, finally, in section 7, we offer a few conclusions regarding the implications of our findings on commodity prices, and some suggestions for future research.

2. Commodity Price Behavior

The econometric analysis of commodity price behavior is based on a variety of generating processes for short-term price movements. Although these often appear completely random, arguments can be made for price regularities or cycles. Models as simple as the cobweb give the most basic explanation of these: in such models, the market clears at every point in time, but prices in the supply equation depend on expectations of future prices. The first-order differential equation describing this phenomenon can yield convergent, divergent, or continuously oscillating solutions. More complex mechanisms give rise to second-order equations, of which the solutions can include not only continuous but also damped or explosive oscillations. Further explanations can be found, for example, in Ackerman (1957), Baumol (1962), or Gandolfo (1985).

Developments of these models specific to commodity markets appear in Brock (1988), Chavas and Holt (1991), and Mackey (1989). Studies which expand this approach to include nonlinearity and chaotic behavior

include Burton (1993), Boldrin and Woodford (1990), and Jensen and Urban (1984).

The detection of regularities has proven difficult for a number of reasons. While visual examination of price movements suggests cyclicity, demonstrating the existence of statistically significant cycles can be difficult. Most of the time-domain research on this point has concentrated on short-term fluctuations. An early study of this type was Working's (1958) investigation of the random walk; he proposed that the continuous flow of many different kinds of information into commodity markets causes frequent price changes which might be nearly random. Still, this model allows for some gradualness of price changes, and thus, some degree of very short-term predictability. Samuelson (1965) further developed this theory by postulating that commodity prices follow a martingale process.

Most of the empirical tests of the random walk and martingale hypotheses look for serial correlation and trends, since both processes require price changes to be independent. Trend deviations from a random walk were first discovered for wheat and corn by Houthakker (1961), and for soybeans by Smidt (1965). Labys and Granger (1970), in performing spectral analysis on futures and also spot price series, found some evidence for a modified random walk process, mostly resembling the martingale process. Stevenson and Bear (1970) and Leuthold (1972) also found departures from random walk using filter rules which revealed positive and negative price dependence, thus casting doubt on the validity of the random walk model.

While most of the above studies are based on linear models, other possibilities have been considered. One example is nonlinear models reflecting chaotic behavior. Drawing upon Houthakker's (1961) analysis of cotton prices, Mandelbrot (1963) replaced Gaussian probability laws with those termed "stable Paretian". His approach represented an attempt to discover orderly behavior within what appeared to be a random series of price fluctuations. It is in this context that Frank and Stengos (1989) investigated the martingale hypothesis, using an approach of Sims (1984) as well as a chaos-based approach. Although Frank and Stengos were not able to reject the martingale hypothesis in a series of standard econometric tests involving daily and weekly silver and gold prices, they did provide correlation-dimension-based evidence of the presence of nonlinear structure. This structure was also confirmed using a similar test on soybeans by Blank (1990), and on silver, copper, sugar, and coffee futures by DeCoster, Labys, and Mitchell (1992).

Other recent studies of nonlinear dynamic process in commodity prices have involved the testing of ARCH and GARCH models. For instance, Yang and Brorsen (1992) use GARCH models for corn, pork bellies, soybeans (including meal and oil), sugar, wheat, and gold daily spot price changes. Results from these models suggest that the variance of the price changes

is not constant. Departing from the linear corn/hog price cycle analysis of Jameson (1983), Chavas and Holt (1991) use a GARCH model, but suggest that the pork market may be characterized by other forms of nonlinear dynamics. Two other recent studies which investigate nonlinear dynamics in commodity prices, but which employ a dynamic programming approach, are those of Deaton and Laroque (1992, 1995).

Commodity price studies in the frequency domain have usually taken a spectral analysis approach. For example, Labys and Granger (1970) analyzed agricultural prices, Gelb (1979) and Parikh (1973) coffee prices, Labys, Elliott and Rees (1971) copper prices, Rausser and Cargill (1970) broiler prices, Weiss (1970) cocoa prices, and Slade (1981) several metal prices, using this method. The hog cycle analysis of Talpaz (1974) was frequency related, but was limited to a Fourier representation. The advantage of the spectral approach is that it permits cycles of different frequencies to be discovered and to be tested statistically by methods based on the fast Fourier transform of a price series. This enables researchers to study not only shorter-term or higher frequencies reflecting random-walk behavior, but also medium and long-term frequencies embodying business cycles and/or growth cycles.

The disadvantage of the spectral approach is that the Fourier transform has difficulty with functions having transient components, that is, components localized in time. Another problem is that the Fourier transform of cyclical information does not provide insights into the phase relationships (leads and lags) between cycles of the same frequency in different series. In this study, we hope to show how such problems can be overcome by combining time and resolution analysis through the use of wavelets.

3. A brief theory of wavelets

A concept that, in principle at least, allows one to obtain local time information, while still providing information in the frequency domain, is that of a *multi-resolution analysis*. The notion seems to have grown out of work in which a signal is subjected to a bank of filters: high-pass filters, low-pass filters, medium-pass filters, *etc.* One can imagine listening to the output of an equaliser on hifi equipment. Listening to only one channel, one hears only a narrow band of frequencies. However, the output does not consist of just a continuous unvarying drone. There will normally be variations in pitch over time, of limited extent, but usually perceptible, and there will be possibly substantial variations in volume. A Fourier analysis would have neither of these features. At any definite frequency there would be a fixed intensity (volume), constant over the whole duration of the signal. And it would be a definite frequency, unable to vary even slightly.

What makes the difference? Mostly, the fact that each *band* passed by a filter does not correspond to a definite frequency, but rather to a range of frequencies. The fundamental *uncertainty principle* excludes absolute precision of information in the time and frequency domains simultaneously. For instance, when we deal with discrete time series, the time resolution cannot be any better than the time between successive observations, and the frequency resolution gives no information about frequencies faster than the frequency of observation or slower than the frequency determined by the overall length of the sample.

This trade-off between information in the time and frequency domains inspires multi-resolution analysis. Much of the material we present below derives from the work of Daubechies (1990, 1992), Grossman and Morlet (1989), and Meyer (1992). More formal presentations of wavelet theory can be found in Beylkin *et al.* (1992), Chui (1992a and b), Combes *et al.* (1991), Meyer (1990), Meyer and Coifman (1991), and Wickerhauser (1994).

With monthly data, say, information on phenomena at frequencies shorter than monthly will not be available. But we can certainly consider *lower* levels of resolution, and look at phenomena characterised by quarterly, or yearly frequencies, or by frequencies commonly thought to be associated with the business cycle. It should be said at once that the concept of frequency cannot be used in its usual precise sense in this context. As pointed out by Priestley (1996), the term “frequency” refers only to sines, cosines, and the exponential function of an imaginary argument. It will therefore be preferable to speak of levels of resolution rather than frequencies.

It is clear that a multi-resolution analysis must be a form of smoothing. High-frequency phenomena (those perceptible only at high levels of resolution) ride along on top of phenomena visible at lower resolution, perturbing with rapid movements the slower, smoother, movements corresponding to lower resolutions. We can imagine taking a signal, and passing it through a low-pass filter, in such a way that only variations corresponding to annual or longer frequencies get through. This will yield a smoothly varying filtered output, corresponding to an annual level of resolution. If we then take into account all the variation cut out by the filter, this variation, typically much less smooth, can be thought of as *details* that can be superimposed on top of the annual level of resolution in order to obtain a more detailed picture at the monthly level of resolution. Finally, one could imagine that a new data source becomes available, providing daily data where before only monthly data were to be had. We could subtract from these daily data the monthly data we had before, and thus obtain another level of details.

Consider the space $L^2(\mathbb{R})$ of square-integrable functions on the real line. Then we may define a multi-resolution analysis of $L^2(\mathbb{R})$ as follows:

1. A multi-resolution analysis of $L^2(\mathbb{R})$ is an increasing sequence $\{V_j\}_{j \in \mathbb{Z}}$ of closed subspaces of $L^2(\mathbb{R})$, with the following three properties:

$$\bigcap_{j=-\infty}^{\infty} V_j = \{0\}, \quad \bigcup_{j=-\infty}^{\infty} V_j \text{ is dense in } L^2(\mathbb{R}); \quad (1)$$

2. for all $f \in L^2(\mathbb{R})$, and for all integers $j \in \mathbb{Z}$, $f(x) \in V_0$ if and only if $f(2^j x) \in V_j$; and
3. there exists a function $\phi(x) \in V_0$ such that the sequence of functions $\{\phi(x - k)\}_{k \in \mathbb{Z}}$ constitutes a Riesz basis of V_0 .

The first of these properties defines the different levels of resolution. We see that there is a double infinity of these. The lowest level, corresponding to $j = -\infty$, admits only constant functions, while the highest level, corresponding to $j = +\infty$, encompasses the whole space $L^2(\mathbb{R})$.

The second condition specifies the relation between successive levels of resolution. Any function that belongs to level j corresponds to a function at any other level, by a simple process of dilation by an integer power of 2.

The last condition means, among other things of mainly technical interest, that, for an appropriate choice of normalisation,

$$\sum_{k=-\infty}^{\infty} \phi(x - k) = 1 \quad \forall x \in \mathbb{R}. \quad (2)$$

Thus the set $\{\phi(x - k)\}_{k \in \mathbb{Z}}$ gives a *partition of unity*, and it is this property that allows certain multi-resolution analyses to be *local*.

Let us take a brief look at the very simplest multi-resolution analysis, associated with the name of Haar. In this, the basic function $\phi(x)$, called the *scaling function*, or, sometimes in more recent literature, the *father wavelet*, is just the indicator function for the unit interval:

$$\phi(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } 0 \leq x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$$

With this choice, it is clear that (2) is satisfied. We see at once that V_0 , being generated by the set of all integer translates of the indicator function, is just the set of all functions that are piecewise constant on all segments of the form $[k, k + 1[$, $k \in \mathbb{Z}$. Thus, at the base level of resolution, we can see nothing inside intervals of this form. All details on a scale less than that given by the distance between any two successive integers is lost at this level of resolution.

The second defining property of a multi-resolution analysis allows us to see what the other levels of resolution look like. For $j = 1$, for instance, we see that the function $\phi(2x)$ belongs to V_1 . This can readily be seen to be just the indicator function for the interval $[0, 1/2]$. Thus V_1 is made up by those functions that are piecewise constant on the intervals of length one half for which one end is an integer. There is just twice as much detail available in such functions. It is clear how this can be extended to arbitrary integers j , positive or negative. A positive j gives more detail than that in V_0 ; a negative j less.

The shortcomings of the Haar analysis are clear. Functions that are piecewise constant are maximally smooth *inside* the segments of constancy, but are not even continuous, let alone differentiable, at the points at which adjoining segments touch. We would clearly be prepared to sacrifice constancy inside the segments in favour of a little continuity at the joins. Another multi-resolution analysis can be defined, for which the scaling function is the so-called *tent* function, defined as follows:

$$\phi(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } 0 \leq x \leq 1 \\ 2 - x & \text{for } 1 \leq x \leq 2 \end{cases}$$

Property (2) is again trivially satisfied, and now we see that V_0 contains functions that are piecewise *linear* on the segments $[k, k + 1[$, $k \in \mathbb{Z}$. Continuity at the joins is now achieved; the discontinuity is in the first derivative.

The idea of the preceding paragraph may be extended in order that V_0 should be made up of functions that are piecewise quadratic, or piecewise cubic, *etc*, and that have discontinuities of the second, third, *etc*, derivative at the joins. Such functions are called *splines*, and their theory is well developed in the numerical analysis literature. Although they could well form the basis for the sort of procedure we will develop here, it turns out that something even better for our purposes is available.

One highly desirable property of the spline wavelets is that of having compact support. The simplest case, with the indicator function as scaling function, has a scaling function with support just $[0, 1]$. In the case of the tent function, the support is twice as long. Generally, there is a tradeoff between regularity, or smoothness, on the one hand, and the extent of the support of the scaling function. Why is a short support desirable? Precisely because this is what makes a multi-resolution analysis *local*. The shorter the support, the more local is the information in the multi-resolution analysis for any given level of resolution. The Haar analysis achieves perfect localisation at the cost of continuity. Other setups give greater regularity at the cost of less perfect localisation.

We need now to specify just where wavelets come into a multi-resolution analysis. Recall from (1) that $V_0 \subset V_1$. It should therefore

be possible to define a subset of V_1 complementary to V_0 , W_1 say, so that V_1 is the *direct sum* of V_0 and W_1 :

$$V_1 = V_0 \oplus W_1. \quad (3)$$

It is normally possible to find a W_1 that is the span of the set of functions $\psi(x-k)$, $k \in \mathbb{Z}$, for some suitable function ψ , called the *wavelet* of the multi-resolution analysis. (It would be the *mother wavelet* in the terminology that calls the scaling function the father wavelet.) In the Haar case, the obvious choice is the following function:

$$\psi(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } 0 \leq x < 1/2 \\ -1 & \text{for } 1/2 \leq x < 1 \\ 0 & \text{for } x \geq 1. \end{cases}$$

It is clearly possible to construct any function that is piecewise constant on all the intervals of length one half and one endpoint an integer by taking a linear combination of the $\phi(x-k)$, that is, the indicator functions on the unit intervals, and of the $\psi(x-k)$. Then these piecewise constant functions make up V_1 in the Haar case.

In addition, from the second two defining properties of a multi-resolution analysis, V_0 is spanned by the functions $\phi(x-k)$, $k \in \mathbb{Z}$, and V_1 is spanned by the $\phi(2x-k)$, $k \in \mathbb{Z}$. Thus it must be possible to express both $\phi(x)$ and $\psi(x)$ as linear combinations of the $\phi(2x-k)$, $k \in \mathbb{Z}$. If in addition ϕ and ψ have compact (bounded) support, these linear combinations are necessarily finite. Thus there are coefficients c_k and g_k such that, for all $x \in \mathbb{R}$:

$$\phi(x) = \sum_{k=0}^K c_k \phi(2x-k), \quad \text{and} \quad (4)$$

$$\psi(x) = \sum_{k=0}^K g_k \phi(2x-k) \quad (5)$$

for some finite K . Equations (4) and (5) constitute what is called the *scaling filter* associated with the multi-resolution analysis. It turns out that the g_k can be chosen in a determined way if the c_k are given, and that in fact the scaling function ϕ and the wavelet ψ are fully determined by (4) and (5).

The *Daubechies wavelets* that we will use in our applications make use of sets $\{c_k\}$ which turn the functions $\phi(x-k)$ and $\psi(x-k)$ into an *orthonormal basis* of the space they span. Thus the Daubechies father and

mother wavelets satisfy the following orthogonality properties:

$$\begin{aligned} \int_{-\infty}^{\infty} \phi(x) \psi(x - k) dx &= 0, \quad k \in \mathbb{Z} \\ \int_{-\infty}^{\infty} \phi(x) \phi(x - k) dx &= 0, \quad k \in \mathbb{Z}, \quad k \neq 0, \\ \int_{-\infty}^{\infty} \psi(x) \psi(x - k) dx &= 0, \quad b \in \mathbb{Z}, \quad k \neq 0, \end{aligned}$$

and the following normalisation properties

$$\int_{-\infty}^{\infty} \phi^2(x) dx = \int_{-\infty}^{\infty} \psi^2(x) dx = 1.$$

It is in fact not too difficult to find wavelets satisfying the orthonormality properties above; there are many such examples. Note, however, that the spline wavelets do not do so, except for the Haar wavelet.

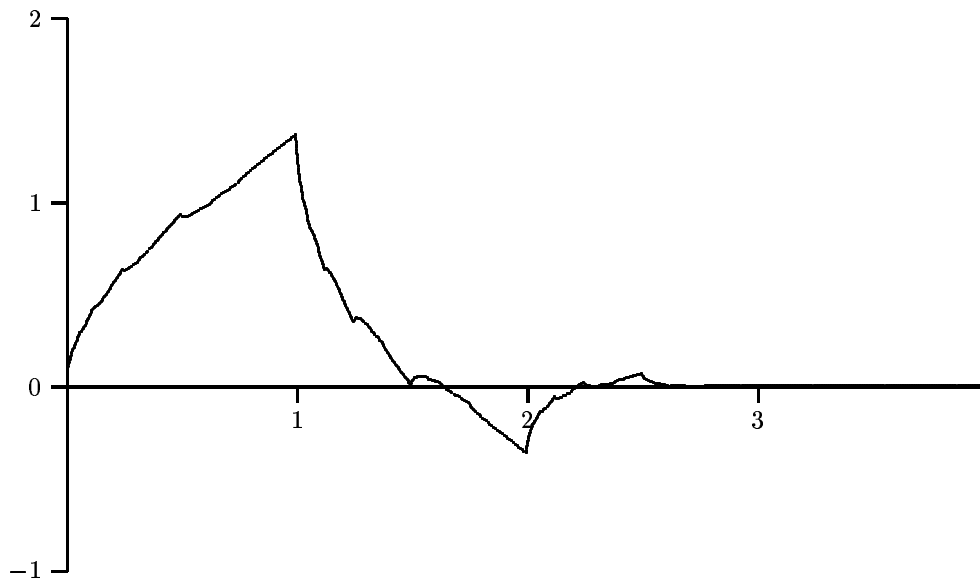
What makes the Daubechies wavelets really useful is that they have compact support, and a certain number of *vanishing moments*. Thus, for each set of Daubechies wavelets, there is an integer m such that, for all integers k with $0 \leq k \leq m$,

$$\int_{-\infty}^{\infty} x^k \psi(x) dx = 0. \tag{6}$$

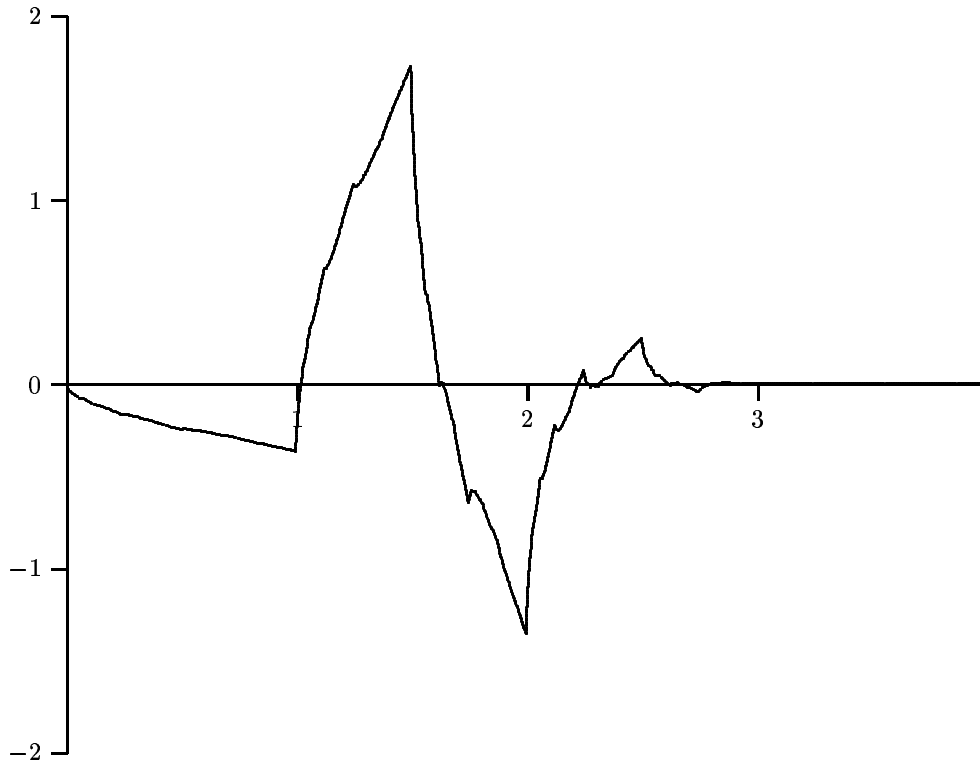
Since ψ has compact support, the effective range of the above integral is not at all infinite. It is easy to see as well that the number of nonzero c_k in the scaling filter is finite.

The tradeoff to be taken account of at this point is between the length of the support of the wavelets and the number of vanishing moments. An increase in m can be bought only at the cost of extending the length of the support, that is, of lessening the degree of localisation achieved by the wavelets. However, a greater value of m also provides a higher degree of continuity and differentiability of the wavelets. The Daubechies wavelets we actually use in this paper have $m = 2$. The graphs of the father and mother wavelets are shown in Figure 1. It can be seen that the father wavelet is not too dissimilar to the tent function, while the mother wavelet is oscillatory, with four main crossings of the axis, although there are many more in the right-hand tail.

Suppose then that we choose a value of m well adapted to our purposes. We may define V_0 as the space spanned by the functions $\phi(x - k)$, $k \in \mathbb{Z}$. Because of (3), V_1 is then spanned by these functions along with the $\psi(x - k)$, $k \in \mathbb{Z}$. As we proceed to successively higher resolutions, the spaces



Daubechies Scaling Function or Father Wavelet



Daubechies (Mother) Wavelet

Figure 1

V_l are spanned by the functions already in use along with the $2^{j/2}\psi(2^j x - k)$, $j < l$, $k \in \mathbb{Z}$. (The factors of $2^{j/2}$ merely serve to normalise the functions.) In this way, we can construct a multi-resolution analysis of $L^2(\mathbb{R})$ based on Daubechies wavelets.

In most applications, we neither need nor want to consider functions defined on the whole real line. If attention is restricted to functions with support in some finite interval $[a, b]$, then the compactness of the support of the Daubechies wavelets means that, at any given resolution, only a finite number of the wavelets making up the orthonormal basis at that resolution have support that intersects $[a, b]$.

Let us suppose, without loss of generality, since we can always rescale and relocate a function with compact support, that the interval we consider is $[0, 1]$. It turns out that the support of a Daubechies wavelet is an interval of the form $[0, 2m + 1]$, where m is the highest nonvanishing moment of the wavelet ψ ; see (6). Let j_0 be the smallest integer such that $2m + 1 \leq 2^{j_0}$. Then the wavelets $\phi(2^{j_0} x - k)$ have support of length $2^{-j_0}(2m + 1) < 1$. Define a set of 2^{j_0} functions $\phi_k(\cdot)$, $0 \leq k < 2^{j_0}$, on $[0, 1]$ as follows:

$$\phi_k(x) = \begin{cases} \phi(2^{j_0} x - k) & \text{if } 2^{j_0} x - k \geq 0 \\ \phi(2^{j_0}(x + 1) - k) & \text{otherwise.} \end{cases} \quad (7)$$

Thus the ϕ_k are just the scaling functions “wrapped round” on to the interval $[0, 1]$ when their supports extend beyond the endpoint 1. Then the functions $\phi_k(x)$ yield a partition of unity on $[0, 1]$:

$$\sum_{k=0}^{2^{j_0}-1} \phi_k(x) = 1, \quad \text{for all } x \in [0, 1]; \quad (8)$$

they are mutually orthogonal, and they can be normalised to have L^2 norm of unity at the cost of making the right-hand side of (8) a power of 2 rather than 1. We may regard them as spanning the subspace of $L^2[0, 1]$ that corresponds to the “base” level of resolution.

Higher levels of resolution are obtained by, first, appending to the set ϕ_k another 2^{j_0} functions ψ_k , defined just like the ϕ_k in (7), but with the mother wavelet ψ in place of the father wavelet ϕ ; then, subsequently, sets of functions of the form $\psi(2^j x - k)$, $j > j_0$, $k = 0, \dots, 2^j$, with the same sort of wrapping round as in (7). It is easy to see that, at each resolution, there are exactly 2^{j+1} basis functions, for each $j \geq j_0$. Given the second defining property of a multi-resolution analysis, another way to span the space of the resolution with 2^{j+1} basis functions, based exclusively on the father wavelet, is to use the functions $\phi(2^{j+1} x - k)$, $k = 0, \dots, 2^{j+1} - 1$, suitably wrapped round.

The *discrete wavelet transform* is constructed on the basis of the above. Let us denote by $\theta_k(\cdot)$, $k = 0, \dots, 2^j - 1$ the functions making up an orthonormal basis of some level of resolution of $L^2[0, 1]$ by use of one set of father wavelets and various sets of mother wavelets. Then consider a function $f \in L^2[0, 1]$. Its representation at this resolution is just the orthonormal projection of f on to the span of the θ_k :

$$f^j(x) \equiv \sum_{k=0}^{2^j-1} a_k \theta_k(x),$$

where

$$a_k = \int_0^1 f(x) \theta_k(x) dx.$$

Similarly, let the same space be spanned by the orthonormal set of functions $\eta_k(\cdot)$, $k = 0, \dots, 2^j - 1$, where the η_k are translations and dilations of the father wavelet only. Then we may write

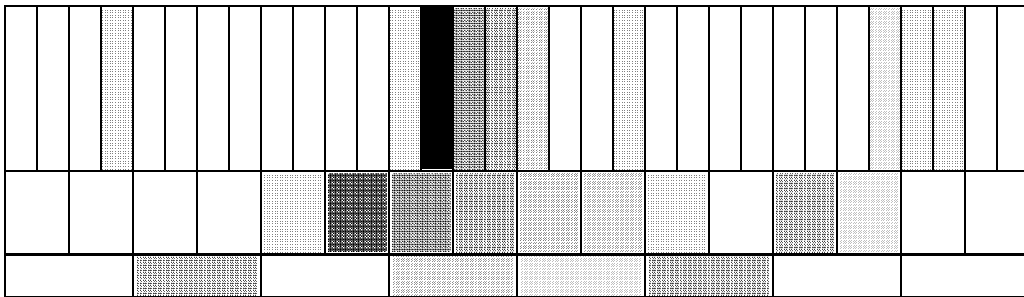
$$f^j(x) = \sum_{k=0}^{2^j-1} b_k \eta_k(x),$$

where

$$b_k = \int_0^1 f(x) \eta_k(x) dx. \tag{9}$$

At this resolution then, the function f is represented by either one of the two 2^j -vectors \mathbf{a} and \mathbf{b} , with typical elements a_k and b_k respectively.

Since the θ_k constitute an orthonormal basis, as do the η_k , the vectors \mathbf{a} and \mathbf{b} corresponding to an given f are related by a matrix orthonormal transformation independent of f . This orthonormal transformation can be implemented by a fast algorithm constructed on the same lines as the fast Fourier transform. It is called the *fast wavelet transform*. Like the FFT, it applies only to vectors with exactly 2^j components, for some positive integer j . The computational details (for the Daubechies wavelets of low order) can be found in Press *et al.* (1992).



A Patio Plot

Figure 2

The interest of the discrete wavelet transform, fast or not, is as follows. The vector \mathbf{b} can be thought of as a *sampling* of the values of the function f , sufficient for a description of f at resolution j . In fact, since the support of $\phi(2^j x - k)$ is the interval $[k2^{-j}, (k + 2m + 1)2^{-j}]$ (ignoring wrap-around), (9) can be thought of as providing (up to a scalar factor) a weighted average of the values of f on this interval. On the other hand, \mathbf{a} provides information about f with respect to time and level of resolution. Unlike a Fourier frequency-domain representation, the representation provided by \mathbf{a} is discrete. Its information content can be displayed visually by means of a “patio” diagram, as illustrated in Figure 2. Each tile of such a patio represents one of the components b_k (the darkness of shading is proportional to b_k^2), and each of these components corresponds to a time interval and a resolution, which are, respectively, the support and the resolution level of the corresponding η_k . As Figure 2 shows clearly, the time intervals are longer at lower resolutions, on account of the uncertainty principle.

4. Wavelet Estimation and Tests

One of the most constraining features of the various wavelet transforms considered in section 3 is that they apply to data sets for which the number of observations is an integer power of 2. In addition, regarding the interpretation of wavelet transforms, there is almost always a presumption that the observations are equally spaced in time.

There exist suggestions in the literature as to how to overcome these constraints. For instance, methods exist for padding data sets so that the number of observations is increased to the smallest power of 2 greater than (or equal to) the size of the observed sample. Similarly, no great difficulties of interpretation arise if the data, rather than being equally spaced, are spaced by random drawings from a known distribution.

If for the moment we ignore these limitations, the method of data analysis known as *wavelet shrinkage* has great success in coping with data sets of widely different natures, where the observations can be regarded as a realisation of some stochastic process. The simplest example is a nonparametric regression, in which successive observations on a random variable are generated by an unknown deterministic function of the time, plus random noise. Formally,

$$y_t = g(t) + u_t.$$

Here y_t is the t^{th} observation on the dependent variable, $g(\cdot)$ is the unknown deterministic function to be estimated, and u_t is the t^{th} element of a zero mean noise process, which in general may be heteroskedastic and/or serially correlated.

In order to perform wavelet shrinkage, the sequence of points y_t , $t = 1, \dots, T$, (after padding so that $T = 2^i$ for some positive integer i) is subjected to a discrete wavelet transform, which will yield a set of transform coefficients associated with different translations and dilations of the mother and father wavelet functions. The “shrinkage” process then either completely eliminates or else greatly reduces all the coefficients smaller in absolute value than some suitable threshold. Finally, the shrunk coefficients are used in order to perform an inverse transform, the result of which is the estimate of the function $g(\cdot)$.

This procedure is advocated in Donoho, Johnstone, Kerkyacharian, and Picard (1995). Many advantages to its credit are pointed out, not only in the paper itself, but in the discussion published along with it. In particular, the estimate of $g(\cdot)$ is in most circumstances “as smooth as the truth”, that is, the unknown function g itself. The authors present examples for which this function has actual discontinuities, which are well captured by the wavelet estimator. Closely related work is presented in Antoniadis (1994), in which various optimality results are proved for a procedure which, while not referred to as “wavelet shrinkage”, nevertheless is clearly a variant of it.

Like all nonparametric methods, this one has an adjustable parameter, here the threshold below which shrinkage takes place. Like most nonparametric methods, a suitable value for the adjustable parameter can often be found by cross-validation; see Nason (1996) for numerous suggestions as to the implementation of cross-validation in this context, and Hall and Patil (1996) for further theoretical discussion of the choice of the adjustable parameter.

For our present purposes, wavelet shrinkage can be applied directly to the price series we consider, since they consist of equally spaced observations, assuming only that we use an appropriate padding method to extend our data series so as to have 2^i observations. However, for the purposes of the nonparametric regression as a tool for econometricians, the constraints mentioned at the beginning of this section are too severe for wavelet shrinkage to be a practical procedure. What to do if there are missing data points? What to do if the independent variable is not the time, but rather an exogenous variable the distribution of which we do not wish to model? What to do when our data have trends, thus making untenable the usual assumption that the unknown function $g(\cdot)$ is periodic, with period the length of the observed sample? Applying the usual procedure blindly leads to strange “edge effects” in the estimates of $g(\cdot)$ near the beginning and the end of the sample. (Antoniadis (1994) alludes to ways around this involving “tapering” of the wavelet series, but he provides no details.)

The suggestion we make in this paper is to use the translations of the scaling function and the translations and dilations of the mother wavelet function in the same way as we use trigonometric functions, or orthogonal polynomials, in semi-nonparametric regression. Thus we select a particular scaling function with its corresponding wavelet, and treat them as functions that we can evaluate for any given argument. Then we can simply regress our dependent variable on a set of translations and dilations of the basic functions evaluated at the observed values of the explanatory variable.

One advantage of this method is that we can apply standard results on the statistical properties of semi-nonparametric regressions directly. Another is that it answers simply and naturally most of the questions posed above. There are no constraints at all regarding the number of observations, and missing observations are just that – they do not appear in the sample. Unequally spaced values of the explanatory variable are handled as easily as equally spaced ones. Edge effects can still occur with trending data, as they can with any semi-nonparametric regression, although for a different reason from the one that applies to the standard wavelet transform. We will see later that a simple trick allows us to reduce these effects to a minimum.

In order to analyse the time series we consider, we proceed as follows. First, we compress the length of the observed series to the $[0, 1]$ interval. Since we treat monthly (and so equally spaced) series, the values of the independent variable at which the various functions appearing in our semi-nonparametric regression are evaluated take the form i/n , where n is the sample size (number of observations), and $i = 1, \dots, n$. For the purposes of this paper, we use the Daubechies wavelet with $m = 2$ (see (6)), that is, a wavelet ψ and a scaling function ϕ (mother and father wavelet) such that the support of both functions is the interval $[0, 5]$. By use of an inverse discrete wavelet transform, ϕ and ψ are evaluated at a fine grid of 2^i points, such that 2^i is much greater than the sample size n . The choice of i simply determines the accuracy with which ϕ and ψ are evaluated for an arbitrary argument x : given x , we take for $\phi(x)$ the value of ϕ at the closest point to x in our grid of 2^i points.

At our base level of resolution, we use the functions ϕ_k defined in (7) for $j_0 = 3$ (since $2m + 1 = 5 < 2^3$). These functions all have support of length $5/(2^3) = 5/8$, and the different ϕ_k are all translates of one another by an integer multiple of $1/8$. In order to take account of all translates that intersect the $[0, 1]$ interval, therefore, we need 12 translates, namely those whose support begins at $-1/2, -3/8, -1/4, \dots, 5/8, 3/4, 7/8$. In order to perform an analysis of a series (of n observations) at this resolution, we now form an $n \times 12$ regressor matrix, the elements of which are the 12 functions evaluated at the points i/n , $i = 1, \dots, n$, on which we regress the $n \times 1$ vector given by the series under analysis. Note that we use 12 different

functions rather than using the “wrap-around” procedure characteristic of the discrete wavelet transform in order to reduce edge effects, which would otherwise be substantial with the strongly trending series we will consider.

At the next stage, we need exactly the same number of translates of ψ at the base resolution, since the support of ψ is the same as that of ϕ . At this next resolution level, we would use an $n \times 24$ regressor matrix. After that, we need $16 + 4 = 20$ translates of ψ shrunk so as to have support $5/16$, thereby obtaining an $n \times 44$ regressor matrix, and then $32 + 4 = 36$ translates of ψ shrunk to support $5/32$, and so on. In fact, we will stop at this last level of resolution. Thus, at our highest resolution level, we need $12 + 12 + 20 + 36 = 80$ functions based on either ϕ or ψ .

How then to interpret the results of these semi-nonparametric regressions, at different resolutions? The first, in which one regresses a series on the 12 translates of the scaling function ϕ , gives a fit that we will interpret as the *trend* of the series. This interpretation is, of course, somewhat arbitrary, in that we choose a specific resolution for what we call the trend. The choice was made on the basis of the appearance of the fits we actually obtained, as described in the following section. Note that the *mean* of the series being analysed is entirely captured by this first regression, since ψ integrates to zero. In addition, since the Daubechies wavelets we use have vanishing first and second moments ($m = 2$), any linear or quadratic time trend in the data is also completely captured. In fact, numerical experimentation, in which cubic and quartic trends were regressed on the 12 regressors of the first stage, shows that, although they are not exactly fit, as linear or quadratic trends are, they are nonetheless fit with a high degree of accuracy. In particular, plots, on paper or a computer screen, of such a trend and its wavelet fit cannot be distinguished by eye.

As we move to higher resolutions, we find at the first stage another 12 regressors, based now on ψ . The shape of the Daubechies wavelets is such that the first four translates, those whose supports begin at $-1/2, -3/8, -1/4, -1/8$, are not very different from zero on $[0, 1]$. They do not seem to play a significant role in our analysis, except for ironing out the edge effects alluded to earlier. We therefore ignore them for our interpretation.

That leave us with 8 translates, which we can associate with the 8 intervals of length $1/8$, beginning successively at $0, 1/8, \dots, 7/8$. We wish to interpret the contribution of each translate to the total fit as a measure of the amount of “activity”, or variation, of the series, at time scale $1/8$ of the total sample period, at the appropriate time, that is, a time interval of the form $[i/8, (i + 1)/8]$, $i = 0, \dots, 7$.

This interpretation would be simple and unambiguous if we had performed a discrete wavelet transform. As we have seen, such a transform

corresponds to a transformation of the data by an orthogonal matrix. But, although the wavelet functions, considered as elements of $L^2(\mathbb{R})$, form an orthonormal basis, the columns of the regressor matrices we use are neither normalised nor orthogonal. This is in general the case for two distinct reasons: the wavelet functions are evaluated at a discrete set of points only, and those whose support extends outside $[0, 1]$ are truncated.

This being the case, the appropriate measure of variation in each time cell seems to be provided by (the square of) the t statistic on the regressor corresponding to that cell. This quantity has of course its usual statistical interpretation, and here it has the twofold virtue, first, that it is invariant to the scale of the corresponding regressor, thereby freeing us from any concern due to the lack of normalisation of these regressors, and, second, that it measures the *marginal* significance of each regressor, thereby making it unnecessary that the regressors be mutually orthogonal. In point of fact, the t statistics associated with the four leftmost translates, at any resolution, are very rarely significant at conventional levels, and the other translates, except the rightmost two or three, are not far from being orthogonal.

Similar interpretations are available at higher resolutions. At the last one we consider, we have 32 (*i.e.* $36 - 4$) time intervals in which we can measure the variation at time scale $1/32$ of the total sample period.

We also wish to be able to compare intensities of variation across different time scales. To this end, we determine the overall contribution of any given time scale by its marginal contribution to the explained sum of squares of the regression. For instance, at the second highest resolution, which corresponds to a time scale of $1/16$ of the sample period, the contribution is proportional to the increase in the explained sum of squares in passing from the regression at the next lower resolution (with 24 regressors) to the regression at this resolution (with 44 regressors). We can also measure the importance of the residual variation, which we naturally interpret as the variation at time scales shorter than that of the highest resolution actually considered. For this we use a sort of R^2 , but one in which we ignore the variation in the trend of the series. Thus our “total sum of squares” is the increase in the explained sum of squares when we go from the first regression, with just 12 regressors, to the final one, with 80.

In summary, then, each resolution level is assigned an overall importance proportional to its contribution to the explained sum of squares, and within that contribution, each time interval is assigned an importance proportional to the square of the t statistic of the regressor corresponding to that interval at that resolution.

This allows us to construct the “patio” plots we mentioned in the preceding section. These have three rows of subintervals, each row spanning the sample period. The first row, which has 8 subdivisions, corresponds to

the first level of “details” beyond the trend. The next row has 16 subdivisions, corresponding to the next smaller time scale, and the third row has 32 subdivisions, corresponding to the smallest time scale we treat explicitly. Such patio plots will be presented in the next section.

5. Data Analysis: Patio Plots

We performed the wavelet analysis described above for twenty-one international commodity price series, covering the period January 1960 until December 1995, except for three (aluminum, gold and petroleum), which begin in January 1970. Most of these can be found conveniently in a secondary source, the UNCTAD (1996) monthly *Commodity Price Bulletin*. The commodities were selected from those traded on exchanges where prices are determined competitively, or on major markets, rather than on the basis of a mark-up. We use monthly data, because they have enough degrees of freedom for the wavelet analysis to be meaningful, while avoiding the high-frequency noise often found in daily or tick observations. All of the series were checked for consistency of definition over time and for possibilities of errors in recording. Prices are in the currency of the market in question – no series were unnecessarily converted to U.S. dollars in order to minimize possible contamination through exchange-rate fluctuations. In addition, nominal prices were used, because these tend to reflect best the emergence of information that may influence a commodity market, rather than real prices, which are more relevant for studying annual terms of trade than physical market adjustments. This choice also avoids possible contamination by a cyclical price deflator. A definition of each of the series we treat is given in the Appendix.

All the price series were tested for seasonality. Because of the international geographic dispersion of commodity markets and of related trading arbitrage, these prices typically do not have a strong or even a weak seasonal component.

Earlier, we discussed the possibility that edge effects might be important in the wavelet analysis we undertake. Most of these appear to be accounted for satisfactorily by the inclusion of four more regressors at each resolution, as discussed in the previous section. Another device appeared to provide additional help in obtaining reasonable fits at the edges of the sample. This device is a straightforward extension of one used in Donoho, Johnstone, Keryacharian, and Picard (1995). The beginning and end of the sample are artificially extended by some number, say m , observations (we set $m = 16$), in the following way. The series, now of $n + 2m$ elements, is:

$$y_m, y_{m-1}, \dots, y_2, y_1, y_1, y_2, \dots, y_{n-1}, y_n, y_n, y_{n-1}, \dots, y_{n-m+1}.$$

This ensures continuity at the edges, and, for all the series treated, seems to be beneficial as regards the fit of the first few “real” elements.

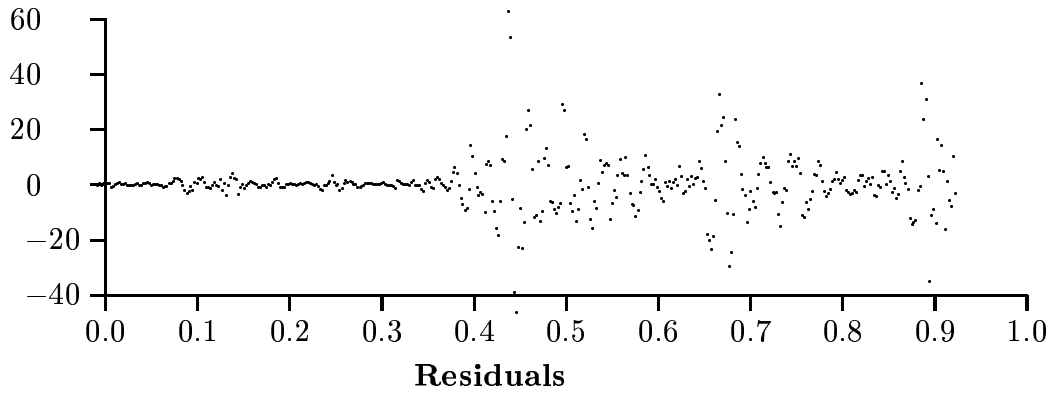
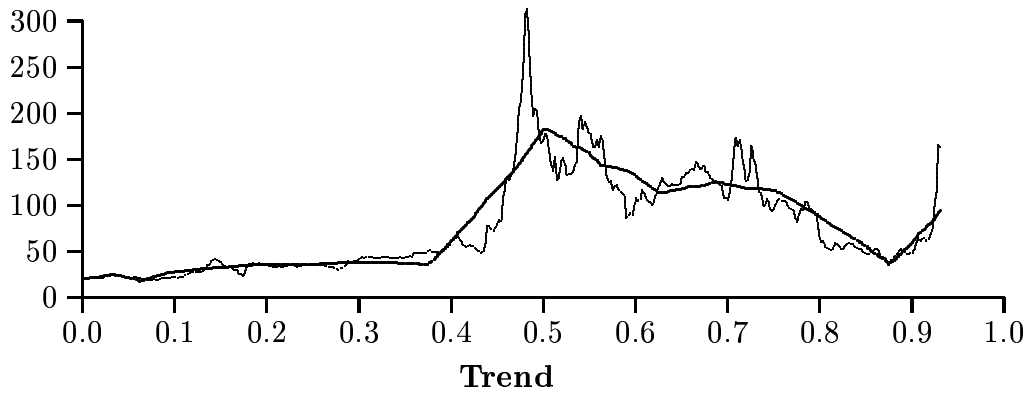
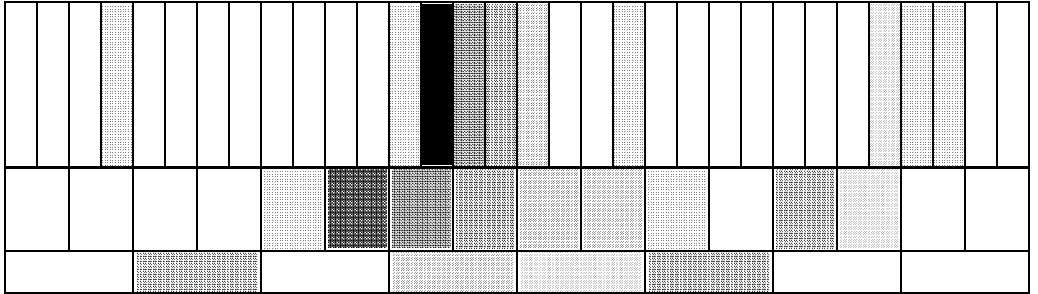
In Figures 3–6 we display graphically the results of the analysis. For four out of the twenty-one commodities considered, namely, coffee, copper, cotton, and rice, we present, first, the patio showing how much movement there is in a series at a certain resolution in a certain sub-interval. (The patio plots for the other 17 commodities are not shown for reasons of space. They may be accessed by anonymous ftp at . . .) At the highest resolution, the sample period is divided into 32 subintervals, each of which is thus roughly a year long. The other, coarser, resolution levels, correspond to time scales of 2 and 4 years approximately.

The graph below the patio in each of the figures shows a plot of the actual series (the finer line) and a plot of the trend, that is, the fit obtained by regressing on the 12 father wavelets at the base level of resolution. Below is a plot of the residuals from the regression on the complete set of 80 wavelet regressors. These residuals represent movement at a level of resolution higher than one year. Since in most cases these residuals are evidently heteroskedastic, it is clear that more information could be obtained by examining the resolutions at 6-month and 3-month time scales. Going further with monthly data would lead in short order to a perfect fit and no degrees of freedom. Lastly, underneath the graphical information, we give the numerical proportions of the total variation accounted for at the various levels of resolution.

The more extensive analysis, down to the 3-month level, was carried out for just one commodity, coffee, and the results are shown in Figure 7. Only two resolutions are shown in this figure, those for which the sample period is subdivided into 64 and 128 subintervals, respectively. These are approximately 6-month and 3-month time scales. For reasons of visibility, the overall patio is now spread out over four lines, each corresponding to one quarter of the length of the sample period. Below, the plot shows the residuals after these two new levels have been accounted for; they may be compared with those of Figure 3.

Coffee

Sample period: 1960.1 until 1995.12

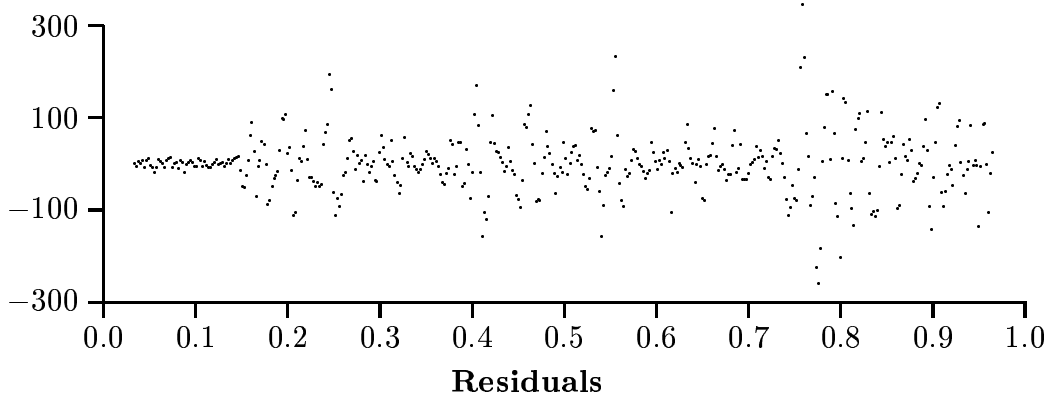
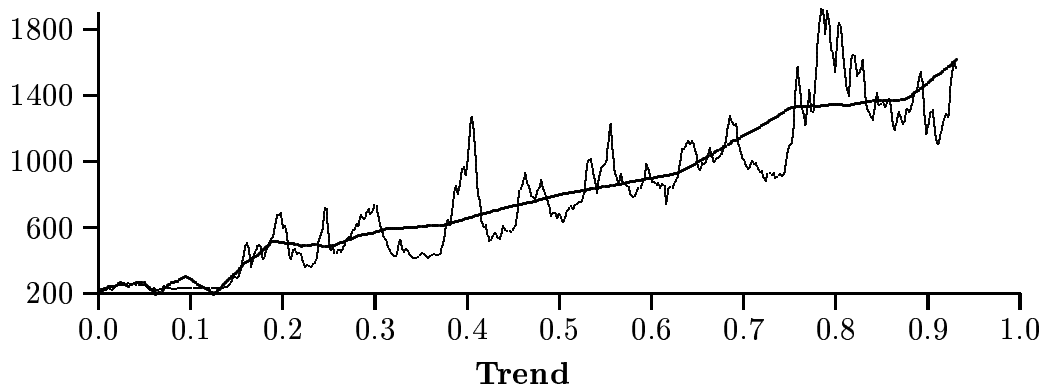
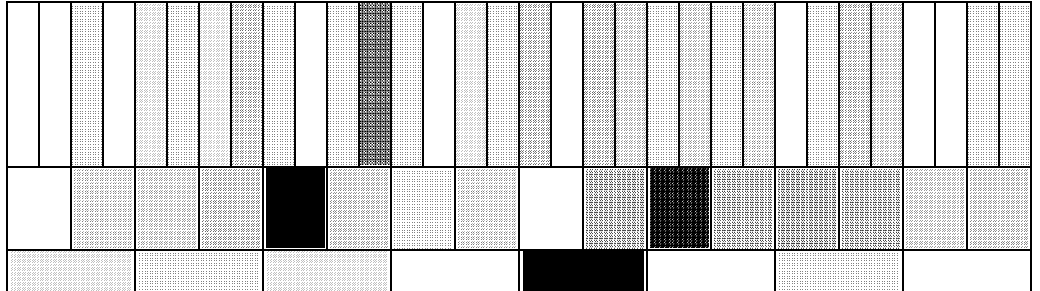


Proportion of variation at resolution 1 = 0.240
Proportion of variation at resolution 2 = 0.455
Proportion of variation at resolution 3 = 0.305
 $R^2 = 0.835$

Figure 3

Copper

Sample period: 1960.1 until 1995.12

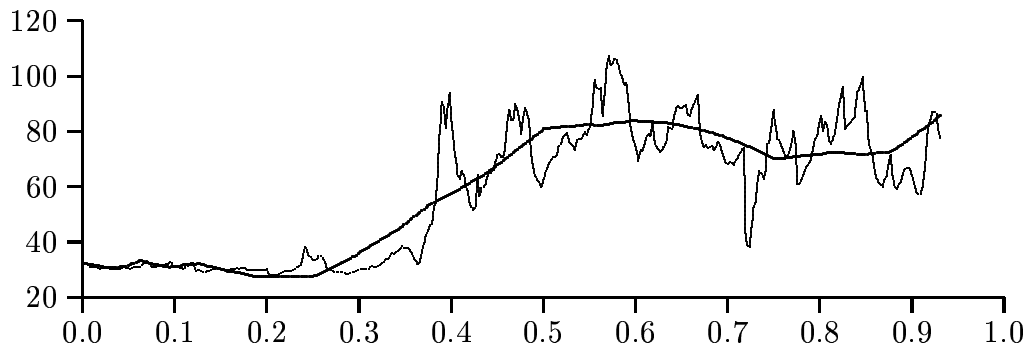
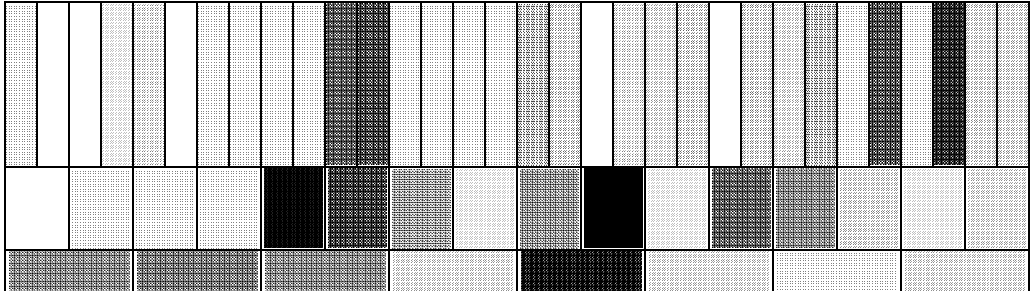


Proportion of variation at resolution 1 = 0.332
Proportion of variation at resolution 2 = 0.532
Proportion of variation at resolution 3 = 0.136
 $R^2 = 0.851$

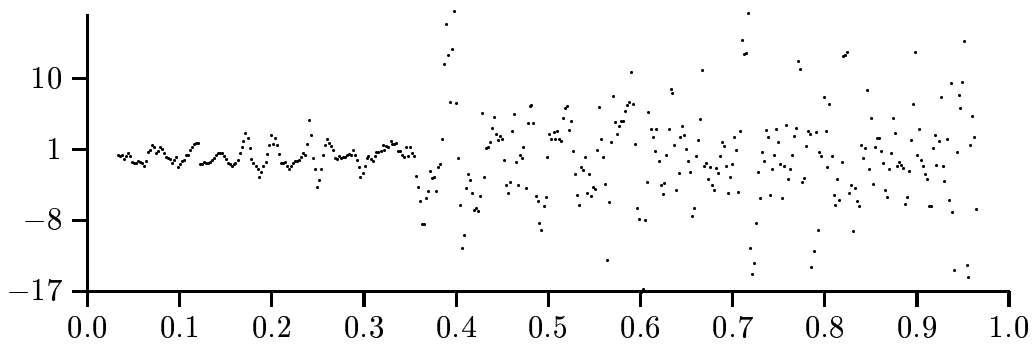
Figure 4

Cotton

Sample period: 1960.1 until 1995.12



Trend



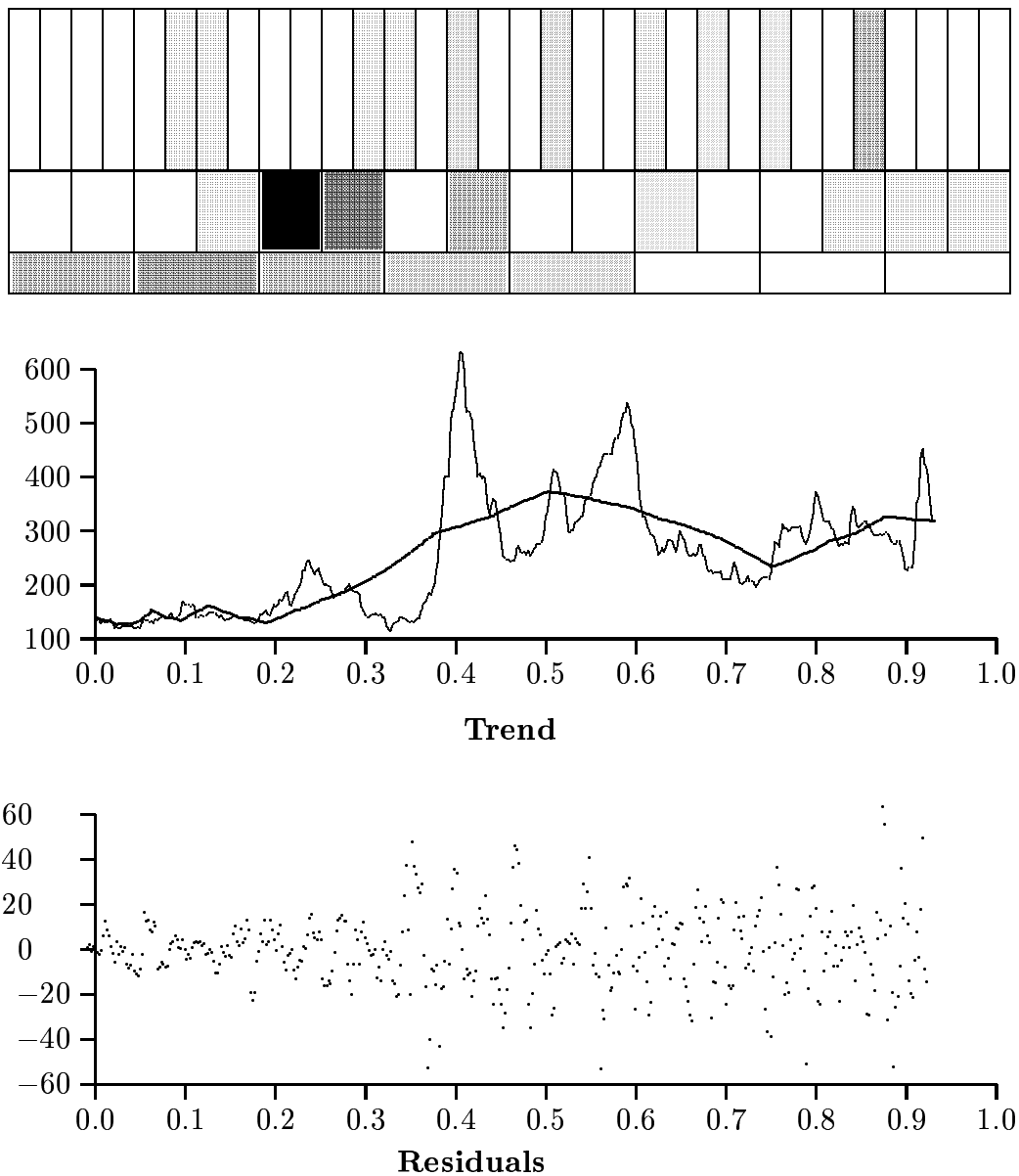
Residuals

Proportion of variation at resolution 1 = 0.353
Proportion of variation at resolution 2 = 0.439
Proportion of variation at resolution 3 = 0.207
 $R^2 = 0.788$

Figure 5

Rice

Sample period: 1960.1 until 1995.12



Proportion of variation at resolution 1 = 0.397
Proportion of variation at resolution 2 = 0.529
Proportion of variation at resolution 3 = 0.074
 $R^2 = 0.946$

Figure 6

Coffee at High Resolution

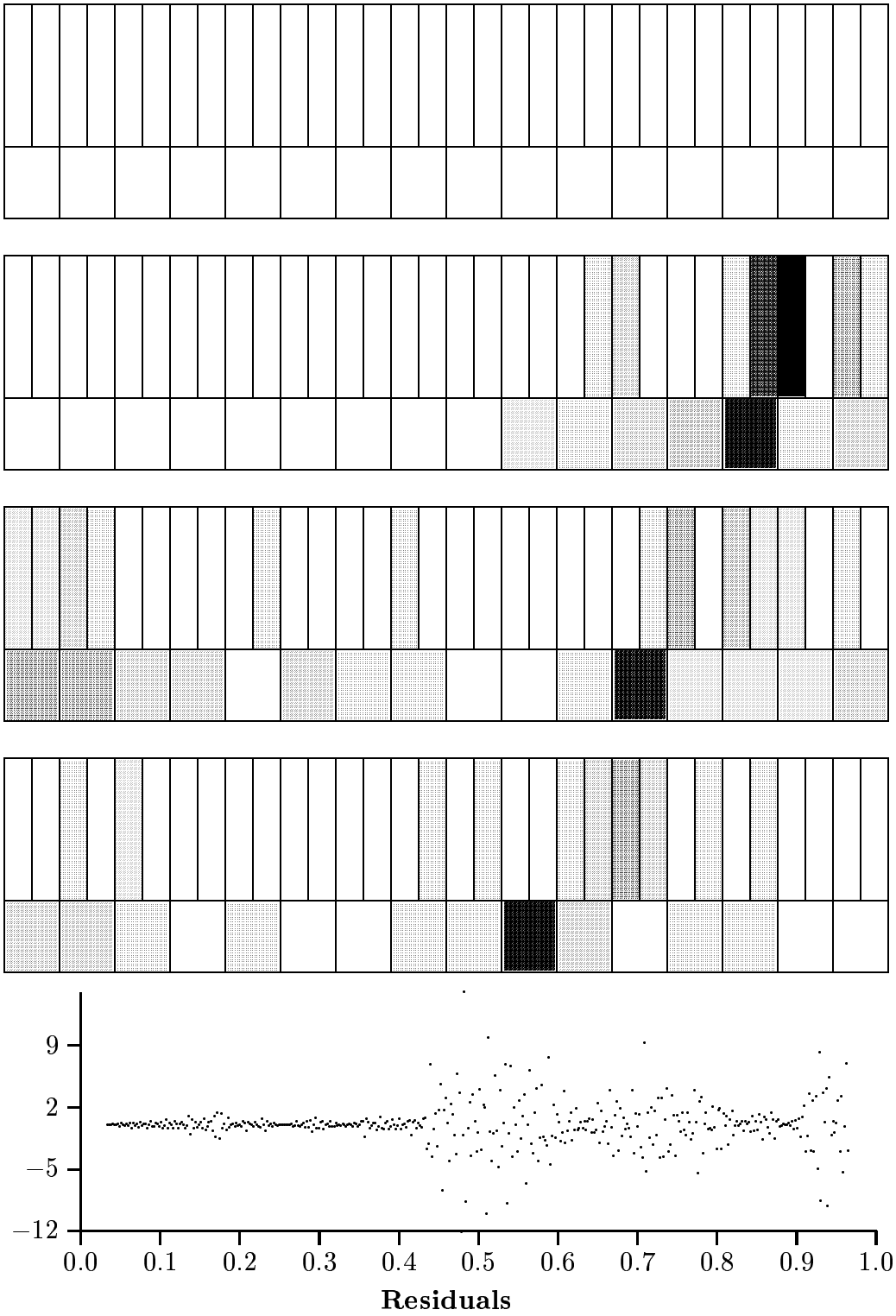


Figure 7

6. Interpretation

The results of the wavelet analyses discussed in the previous section, in particular the patio plots, present us with a number of opportunities to learn how commodity prices behave both as regards time location and time scale.

Price cycles are often considered to display growing volatility over time. If so, then we expect to see darker tiles for later times at the resolution corresponding to the underlying cyclical behavior. If the time scale of the cyclical behavior itself changes, then we expect to see the darker tiles shift from one resolution to another. If cyclical behavior is present at more than one time scale, then dark tiles should be found at more than one resolution. One may well observe that volatility increases over time for some time scales but decreases for others.

A shock, such as the oil shocks that clearly have a strong influence on the series we study here, gives rise to effects that may be more or less persistent. Such effects can be studied not only as regards their persistence in time but also as regards the time scale at which they operate.

If detrending is an issue, the wavelet analysis allows us to distinguish clearly between what we may call short and long run trends, provided only that we are prepared to specify the time scales that we qualify as short or long run. If we suspect the existence of structural breaks, then we would normally try to detect these by examining outliers. Just which observations are outliers, in the sense of being associated with large residuals, may depend on the finest level of resolution used. By comparing different resolutions, we may glean information concerning the nature of possible structural breaks.

At relatively fine levels of resolution, it is possible to see how some commodity prices lead or lag others by seeing when the tiles are darkest for the various price series. One price can be thought of as leading another if its dark tiles consistently occur before those of the other. On the other hand, if most commodities have dark tiles at the same points in time, then we may conclude that the prices of these commodities usually move together.

Wavelet analyses *may* help in forecasting commodity price movements. But one must be wary here, since the very fact of the superior time localization provided by the compact support of wavelets means that no information is available outside of that support.

For the rest of this section, we look in a little more detail at the properties of the twenty-one series analyzed in the light of the above remarks, bearing in mind that our ability to interpret the patios is still rudimentary. First, some general observations. Considerable activity is observed in all the series at the annual and next above resolution during the market

upheaval associated with the petroleum crises. Particularly between 1973 and 1980, one can observe increased volatility in commodity price fluctuations, though some calming appears at the end of this period. The relative shading of the tiles suggests that the intensity of the volatility is greater at different resolutions for different commodities. The fact that activity increases at the 2-year resolution as well as at the annual indicates that the effects of the upheaval over this period were more than transitory – price swings persisted for more than two years before market forces produced a dampening effect. On the other hand, activity at the 4-year resolution appears normally to be more pronounced at the beginning of the period. This is probably due to the strong upward trend in prices starting late in the 1960's and peaking in 1974 or early 1975. At that point, commodity prices were higher than at any time since World War II.

The above general observations change only slightly when we examine the patios of individual commodities. If we begin with the metals, 2-year activity appears strongly for copper, gold, lead, tin, tungsten, and zinc. In time, the first or the fifth tile is the most important for most metals. At the 4-year resolution, the metals differ among themselves. The upward trend in prices occurred earlier for tin, gold, silver, and tungsten than for copper or zinc. Copper, lead, and zinc also display similar patterns at 1-year resolution, whereas the periods of greatest activity for tin come later. This perhaps reflects the instability caused by the collapse of the tin buffer stock and the temporary suspension of tin trading on the LME.

The agricultural commodities are more dissimilar, according to their patios. This is to be expected, since differences exist in their patterns of production and consumption. For example, wheat, corn, and rice are grains grown annually. Coffee, cocoa, and tea are derived from perennial crops in order to produce beverages. Cotton, jute, rubber, and wood are raw materials. In their case, fluctuations in derived or industrial demand affect the behavior of their prices.

If we first consider the grains, it appears that corn and wheat display strong movements at the four-year resolution, and this occurs early in the sample period. This may reflect the fact that wheat was one of the commodities where upward price movements actually preceded the petroleum price jump. Soybean and wheat prices were particularly volatile during the market upheavals of the mid-seventies.

Movements at a resolution of two years are prominent for coffee, cocoa, sugar, and tea, as well as for rice, soybeans, and wheat. All of these except tea display volatility at the annual level, though once again it comes earlier for soybeans and wheat. This displacement could be due to their price volatility being caused by climatic disturbances.

The raw materials, including cotton, jute, rubber, and wool show greater price activity at the longer-run or four-year resolution. This is an important result, since it confirms the relatively greater influence on demand for them of business cycles. For petroleum, the greatest activity at the longer-run resolution comes early in the sample period, reflecting the jump in price trend caused by the imposed price shock. Volatility in short-term price movements comes later, reflecting the troublesome interval in which oil prices fluctuated downwards.

Our graphical results show that commodity price volatility has not grown over time at any but the shortest of time scales and not for all commodities. Annual movements vary in intensity over the sample period, but they do not increase towards the end of the sample. At longer time scales, price movements do not seem to be most intense at any particular frequency. The darkest tiles are to be found at different time scales at different periods of the sample. The residual plots, however, show that volatility at resolutions of less than a year does increase markedly for almost all the commodities just after the oil shocks. Outliers occur often in the upheaval period of 1973–76, but they seem to have only short-term consequences. Lastly, differences do exist in the time locations of the most intense price activity for different commodities. Leads and lags among periods of intense activity can accordingly be observed, as we saw with tin as opposed to copper and zinc. If this observation can be generalized, it would lend support to the traditional view that commodity prices do not move together, a view that has recently been questioned.

7. Conclusions

In this paper, we have developed a semi-nonparametric approach to data analysis, based on wavelets, that to our knowledge has not been applied in other contexts. We make use of the fact that the father and mother wavelets used in multi-resolution analysis are simply new special functions, with properties that make them particularly attractive for many purposes. This fact allows us to overcome many of the limitations of the wavelet transforms conventionally used in signal analysis. The fast wavelet transforms that have been developed in recent years make it simple to compute the values of these new special functions for any argument we wish. The regressors used in the semi-nonparametric regressions that gave rise to all the results in this paper are just vectors whose elements are such values of the wavelet functions. There is no restriction on the number of elements in these vectors, and there is no requirement of equal spacing.

The results obtained here indicate that our approach has considerable potential when applied to economic data. In particular, the patio plots

provide valuable information on both the time location and the time scale of price movements. Since this wavelet-based methodology is new, the interpretation of the results it provides is as yet tentative. However, because of the international importance of commodity price behavior, econometric results are readily available from other sources, and these have helped us to interpret the numerical results we obtain. In particular, the wavelet analysis seems effective in the analysis of several complex problem areas which exist in price analysis. Among these, we may cite:

- (1) the tendency towards greater volatility or heteroskedasticity;
- (2) the effect of random shocks in giving rise to outliers;
- (3) the impact of market conditions in giving rise to structural breaks; and
- (4) the possible common influence of economic events causing commodity prices to move or not to move together.

A great many issues await further research. We have paid very scant attention to the treatment of edge effects – what worked well enough here may not continue to do so in other contexts. A related issue is the precise choice of the time scales corresponding to the different resolutions. Here we took the entire sample period and divided it into eight subintervals. It would have been possible to choose the year as the base subinterval instead of the approximate year used here, and it would be interesting to see to what extent our results would change if we did so. The Daubechies wavelet with $m = 2$ works well enough for our purposes, but other choices seem equally reasonable. These should be investigated. The fact that our approach uses a semi-nonparametric regression means that standard inference procedures can be used for hypothesis testing. So far, we have done none of that. Even so, a little further knowledge has been added to our understanding of commodity price behavior.

Appendix

Data Description and Sources

Aluminum

London Metal Exchange, high grade, cash. From February 1970 to December 1978: virgin ingot, 99.5% purity, c.i.f. Europe. London (*Metal Bulletin*, London), 1970.01–1995.12.

Cocoa

Average of daily prices of the nearest three active future trading months on the London Terminal Market and on the New York Coffee, Sugar, and Cocoa Exchange at time of the London close. Article 26 of the International Cocoa Agreement, 1986 (International Cocoa Organization, London), 1960.01–1995.12.

Coffee

Average of daily prices (secretariat of the International Coffee Organization, London). Robustas, weighted average of ex-dock New York (60 percent), Angola Ambriz 2 BB, Uganda standard, 1960.01–1995.12.

Copper

London Metal Exchange, electrolytic wire bars, high grade, cash (*Metal Bulletin*, London), 1960.01–1995.12.

Cotton

Medium: U.S. Memphis Territory (medium staple), Middling 1-3/32. Prior to July 1981: S.M. 1-1/16 (USDA, Washington, DC), 1960.01–1995.12.

Gold

United Kingdom, 99.5% fine, London afternoon fixing, average of daily prices (*Metal Bulletin*, London), 1970.01–1995.12.

Jute

Raw Bangladesh, B.W.D., f.o.b. Chittagong-Chalna, actual market prices (*The Public Ledger*, Watford, United Kingdom). Prior to March 1980: minimum export price (Bangladesh Ministry of Jute), 1960.01–1995.12.

Lead

London Metal Exchange settlement and cash seller's price in warehouse excluding duty, range main United Kingdom ports; Purity 99.97% Pb (*Lead and Zinc Statistics*, International Study Group, London), 1960.01–1995.12.

Maize/Corn

U.S. Yellow, No. 3, Average Cash Price, Chicago (USDA), 1960.01–1995.10.

Crude Petroleum

Average of Dubai, United Kingdom Brent, and Alaska N. slope crude prices, reflecting relatively equal consumption of medium, light, and heavy crudes worldwide. Dubai Fateh 32 API, spot, f.o.b. Dubai; United Kingdom, Brent Bland 38 API, spot, f.o.b. United Kingdom ports; United States, Alaskan N. slope 27 API, spot, f.o.b. U.S. Gulf of Mexico ports, 1970.01–1995.12.

Rice

Thailand. White, 5% broken, end of month price, f.o.b. Bangkok, including export duty (IMF Secretariat, Washington, DC), 1960.01–1995.12.

Rubber

Singapore, f.o.b. in bales, No. 1 RSS. closing quotations (*The Public Ledger*, Watford, United Kingdom), 1960.01–1995.12.

Silver

Handy and Harman, 99.9% grade refined, average of daily quotations, New York (*Metal Bulletin*, London), 1960.01–1995.12.

Soybeans

U.S. Yellow, No. 1, Average Cash Price, Chicago (USDA, Washington, DC), 1960.01–1995.07.

Sugar

International Sugar Agreement, daily prices (f.o.b. Caribbean ports, bulk basis) calculated in accordance with article 61 of the International Sugar Agreement, 1977 (International Sugar Organization, London), 1960.01–1995.12.

Tea

London, auction prices, all tea (*Monthly Statistical Summary*, International Tea Committee, London). 1960.01–1995.12.

Tin

Ex-works price Kuala Lumpur market (ITC reference price since July 4, 1972). Tin trade was suspended from October 24 to end of January 1986. (*Metals Week*, New York), 1960.01–1995.12.

Tungsten

Wolfram, c.i.f. European ports concentrates, basis minimum 65% WO_3 (*Metal Bulletin*, London), 1960.01–1995.12.

Wheat

U.S. No. 2, Hard Red Winter (ordinary), f.o.b. Gulf (International Wheat Council), 1960.01–1995.12.

Wool

UK64's (dry-combed basis) (New Zealand Wool Marketing Corporation, Clacton-on-Sea, United Kingdom), 1960.01–1995.12.

Zinc

London Metal Exchange, settlement and cash seller's price in warehouse excluding duty, range main United Kingdom ports; Virgin zinc, high grade (*Lead and Zinc Statistics*, International Study Group, London), 1960.01–1995.12.

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